Developing Meaning for Fractions

Students first learned about numbers through learning the whole number counting sequence and through counting objects. When they encounter fractions, they encounter a different kind of number—numbers that extend the number system by defining numbers between whole numbers. Adults are used to seeing and ascribing meaning to fractions such as $\frac{1}{2}$ and $\frac{3}{4}$ in a variety of situations, but young students need to expand what they understand numbers to be beyond a way to quantify a number of whole things. Imagine how strange fraction notation—two whole numbers separated by a line—must look to elementary school students as they begin to dig into the meaning of fractions.

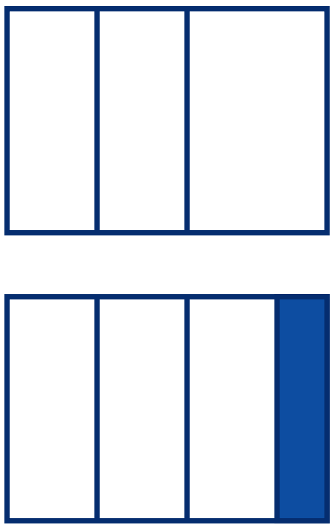
It is not surprising, then, that in trying to understand fractions, students attempt to draw on what they know from their experience with the numbers with which they are most familiar—whole numbers. At first, students may think of "one-half" or "one-third" as a label for a part of a whole. Just as one of the pattern block pieces can be called "red" or "trapezoid," it is also called "one-half" and has the label "$\frac{1}{2}$." They look at the numerator and the denominator as separate whole numbers and are not, at first, thinking about the relationship between the two. For example, students may think that $\frac{1}{3}$ is greater than $\frac{1}{2}$ because 3 is greater than 2.

Looking at fractions as though they represent two separate whole numbers leads to misinterpretation of their meaning and an inability to assess the reasonableness of calculation results. For example, an often-cited assessment question from the National Assessment of Educational Progress (NAEP) asked middle-grade students to estimate the sum of $\frac{12}{13}$ and $\frac{7}{8}$. Given four answer choices—1, 2, 19, and 21—students most often chose 19 or 21. Using whole-number addition, they added either only the numerators or only the denominators. These students did not think of each of these numbers as being close to 1 and thus missed the correct estimate of 2.

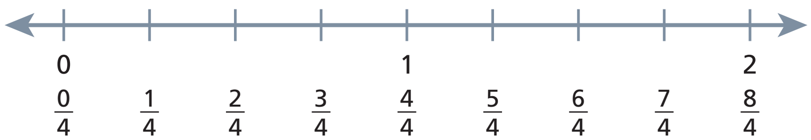
Students develop meaning for fractions in this unit through work with two different and complementary ideas—fractions as equal parts of a whole or several wholes, represented by area representations, and fractions as both distances and numbers represented on a number line.

Students are introduced to three tools they use to learn about fractions as parts of a unit whole. They learn about fractions of objects or area as they use rectangular "brownies," Fraction Sets cut out of rectangles, and pattern blocks. Students use these tools to split the area of the shape into equal parts that represent unit fractions. In this context, "$\frac{1}{2}$" means "one out of two equal parts that make up one whole."

One half of one whole is not the same quantity as one half of another whole; for example, $\frac{1}{2}$ a class of 26 is 13 students, and $\frac{1}{2}$ a class of 22 is 11 students. However, although $\frac{1}{2}$ can represent many different quantities, depending on the size of the whole, $\frac{1}{2}$ has the *same relationship* to *any* whole. It is one of the two equal parts that compose the whole. In this unit, students work with fractions in relation to a whole that is a single object ($\frac{1}{4}$ of one brownie), an area ($\frac{2}{3}$ of the surface of a hexagonal pattern block), or a distance ($\frac{3}{4}$ of the distance from 0 to 1 on a number line). The focus is twofold: the parts of the whole must be equal to one another, and all the parts combined must equal the whole. It is not unusual for third graders to ignore one or both of these ideas at first. For example, when dividing brownies, they may make unequal pieces (as in the first picture) or cut off part of the whole in order to make the pieces equal (as in the second picture below).



The number line is another important tool for understanding the meaning of fractions. It provides a model that has critical differences from dividing objects or area into equal parts. When students divide up objects, as in the *Fraction Cookie* game or the "brownies" problems, they think of *each* whole object as being divided into equal parts. For example, they see that each object has four fourths and they name each part with the unit fraction $\frac{1}{4}$, but they may not see how fractions can represent *more than* four fourths. If they cut up a second brownie, they see another four fourths; they are not necessarily thinking that they now have a total of $\frac{8}{4}$. The context of the ant traveling along the continuous path of the number line opens up the meaning of fractions with numerators greater than 1. If the ant moves $\frac{1}{4}$ of a "block" (defined as the distance between two whole numbers) each time, then rests, the ant stops at $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and $\frac{4}{4}$. It adds on distance and moves to a greater number each time.



By moving past 1, students learn how fractions continue to increase: $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$, and so on, and how the unit fraction $\frac{1}{4}$ is iterated to construct all of these fractions with the same denominator. While it seems to those experienced with the meaning of fractions that fractions less than, equal to, and greater than 1 is consistent, Grade 3 students need time and experience to coordinate what they may see as quite different types of fractions—how, for example, $\frac{1}{4}$, $\frac{4}{4}$, and $\frac{7}{4}$ are related.

Students also work on the idea that fractions greater than 1 are constructed from unit fractions when they share several brownies, e.g., sharing seven brownies equally with four people. As they construct equal portions and then name those portions, they consider how fourths are combined to create fractions greater than 1, and how $1 + \frac{1}{2} + \frac{1}{4}$ and $1\frac{3}{4}$ and $\frac{7}{4}$ model the same quantity.

As students build their understanding of fractions as parts of a whole and fractions as numbers, they are also learning to coordinate the two ideas. In later grades, they will continue to add layers of complexity to the meaning of fractions as they consider fractions as representing division, rates, and ratios.